

# LINEAR PROGRAMMING

KEY

## 1) SKI PROBLEM

A ski manufacturer makes two types of skis and has a fabricating department and a finishing department. A pair of downhill skis requires 6 hours to fabricate and 1 hour to finish. A pair of cross-country skis requires 4 hours to fabricate and 1 hour to finish. The fabricating department has 108 hours of labor available per day. The finishing department has 24 hours of labor available per day. The company makes a profit of \$40 on each pair of downhill skis and a profit of \$30 of each pair of cross-country skis. Find the number of each type of skis that need to be sold to maximize the profit. What is the maximum profit?

Step 1: Define the variables

Let  $x = \#$  of pairs downhill skis

$y = \#$  of pairs cross-country skis

Step 2: Write the objective function

$$P(x, y) = 40x + 30y$$

Step 3: Organize the information (To write the constraints)

What are the factors that limit profit?

	downhill(x)	Cross-country(y)
Fab dept.	6	4
Finish dept.	1	1

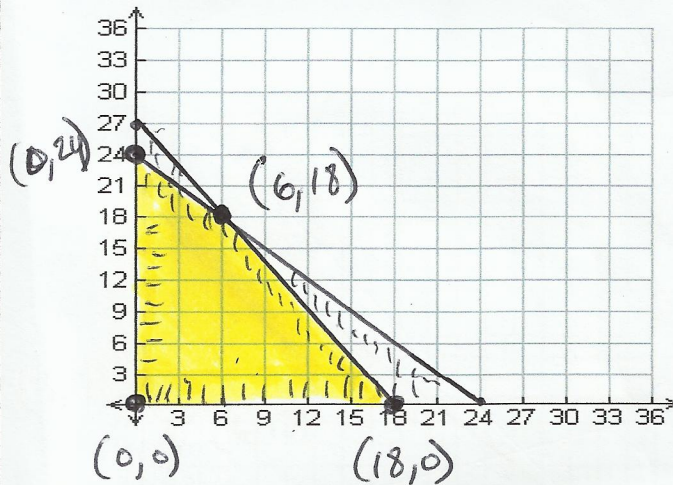
Step 4: Write the constraints

Fabricating:  $6x + 4y \leq 108$

Finishing:  $x + y \leq 24$

# of skis can't be below zero  
 $x \geq 0$   
 $y \geq 0$

Step 5: Graph the feasible region



Step 6: Identify the critical points and find the maximum/minimum of the objective function

$$P(x, y) = 40x + 30y$$

$$P(0, 0) = 40(0) + 30(0) = 0$$

$$P(0, 24) = 40(0) + 30(24) = 720$$

$$P(6, 18) = 40(6) + 30(18) = 780$$

$$P(18, 0) = 40(18) + 30(0) = 720$$

Step 7: Write the answer in a complete sentence

Maximum profit is reached, at \$780, by manufacturing 6 pairs of downhill skis and 18 pairs of cross-country skis.

## 2) MUSIC SHOP

Pinky Lloyd opens a music shop in which he will sell guitars and basses. He wants to find out the maximum amount of money he may have to borrow to purchase the instruments. Each bass will cost him \$900 and each guitar will cost \$750. Pinky has certain restrictions on the numbers of each kind of instrument he can stock. He can buy no more than 48 instruments total. The number of guitars must be at least three times the number of basses. To get started, he must buy at least 18 guitars and at least 3 basses. What number of basses and guitars would give the maximum feasible cost? What would this cost be?

Graph Scale = 3 (label axes so each box is 3 units)

Step 1: Define the variables

$$\begin{aligned} \text{let } x &= \text{basses (\# of)} \\ y &= \text{guitars (\# of)} \end{aligned}$$

Step 2: Write the objective function

$$C(x, y) = 900x + 750y$$

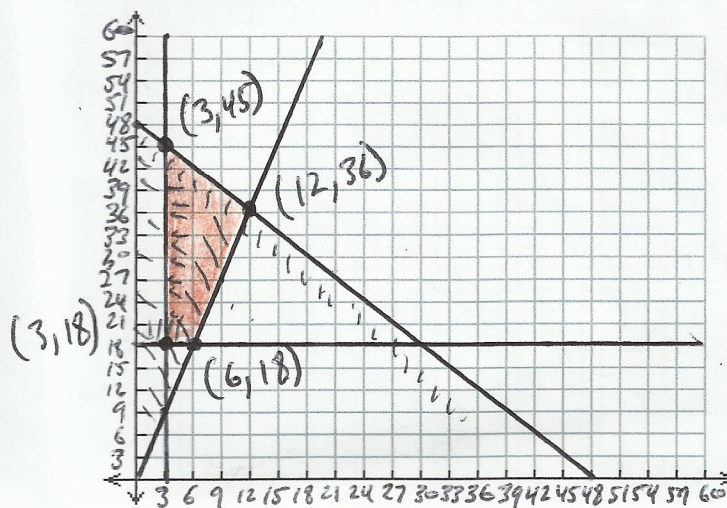
Step 3: Organize the information

guitars + basses  $\rightarrow$  no more than 48  
guitars at least 3 times basses  
at least 18 guitars  
at least 3 basses

Step 4: Write the constraints

$$\begin{aligned} x + y &\leq 48 \\ y &\geq 3x \\ y &\geq 18 \\ x &\geq 3 \\ x &\geq 0 \\ y &\geq 0 \end{aligned}$$

Step 5: Graph the feasible region



Step 6: Identify the critical points and find the maximum/minimum of the objective function

$$\begin{aligned} C(x, y) &= 900x + 750y \\ C(3, 18) &= 900(3) + 750(18) = 16,200 \\ C(3, 45) &= 900(3) + 750(45) = 36,450 \\ C(12, 36) &= 900(12) + 750(36) = 37,800 \\ C(6, 18) &= 900(6) + 750(18) = 18,900 \end{aligned}$$

Step 7: Write the answer in a complete sentence

Maximum cost, at \$37,800, would be reached if he bought 12 basses and 36 guitars.

### 3) EDUCATIONAL COSTS

The school board is investigating various ways of composing the faculty for a new elementary school. They can hire teachers and aides. The board finds that the average teacher's annual salary is \$35,000 and the average aide's annual salary is \$20,000. The building can accommodate no more than 50 total faculty members. A minimum of 20 faculty members is needed to staff the school. There must be at least 5 teachers. The number of teachers must be at least four times the number of aides. Find the number of aides and teachers that would result in the minimum cost for the school board.

Scale = 2

Step 1: Define the variables

$$\begin{aligned} \text{let } x &= \text{teachers} \\ y &= \text{aides} \end{aligned}$$

Step 2: Write the objective function

$$C(x, y) = 35,000x + 20,000y$$

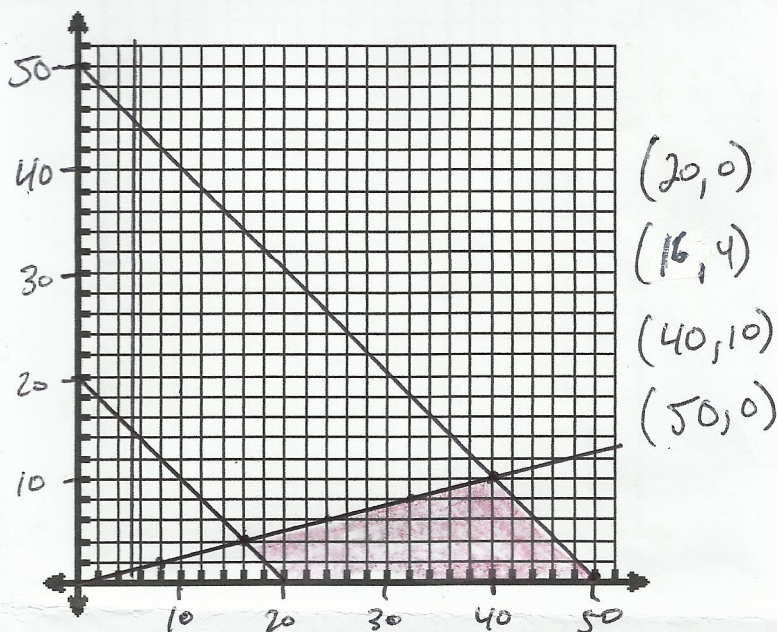
Step 3: Organize the information

$$\begin{aligned} \text{total faculty} &\leq 50 \\ \text{total faculty} &\geq 20 \\ \text{teachers} &\geq 5 \\ \text{teachers} &\geq 4 \text{ times aides} \end{aligned}$$

Step 4: Write the constraints

$$\begin{aligned} x + y &\leq 50 \\ x + y &\geq 20 \\ x &\geq 5 \\ \star x &\geq 4y \rightarrow y \leq \frac{1}{4}x \\ \text{Careful!} \\ x &\geq 0 \\ y &\geq 0 \end{aligned}$$

Step 5: Graph the feasible region



Step 6: Identify the critical points and find the maximum/minimum of the objective function

$$\begin{aligned} C(x, y) &= 35,000x + 20,000y \\ C(20, 0) &= 35,000(20) + 20,000(0) = 700,000 \\ C(16, 4) &= 35,000(16) + 20,000(4) = 640,000 \\ C(40, 10) &= 35,000(40) + 20,000(10) = 1,600,000 \\ C(50, 0) &= 35,000(50) + 20,000(0) = 1,750,000 \end{aligned}$$

Step 7: Write the answer in a complete sentence

The minimum cost can be achieved by hiring 16 teachers and 4 aides. The cost would be \$640,000 / year.

#### 4) PART-TIME JOBS

Rosalyn works no more than 20 hours a week during the school year. She is paid \$10 an hour to tutor geometry students and \$7 an hour for delivering pizzas. She wants to spend at least 3 hours, but no more than 8 hours a week tutoring. Find Rosalyn's maximum earnings and how many hours she would need to work each job.

Scale = 1

Step 1: Define the variables

$$\begin{aligned} \text{let } x &= \text{tutoring hours} \\ y &= \text{pizza hours} \end{aligned}$$

Step 2: Write the objective function

$$P(x, y) = 10x + 7y$$

Step 3: Organize the information

$$\begin{aligned} \text{tutoring} &\geq 3 & \text{tutoring} + \text{pizza} &\leq 20 \\ \text{tutoring} &\leq 8 \end{aligned}$$

Step 4: Write the constraints

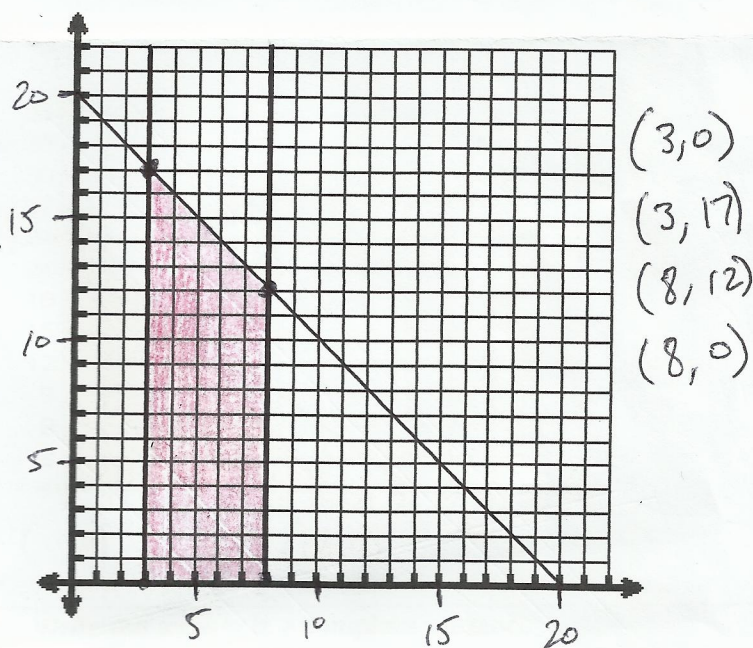
$$x + y \leq 20$$

$$x \geq 3$$

$$x \leq 8$$

$$y \geq 0$$

Step 5: Graph the feasible region



Step 6: Identify the critical points and find the maximum/minimum of the objective function

$$P(x, y) = 10x + 7y$$

$$P(3, 0) = 10(3) + 7(0) = 30$$

$$P(8, 12) = 10(8) + 7(12) = 164$$

$$P(8, 0) = 10(8) + 7(0) = 80$$

Step 7: Write the answer in a complete sentence

Rosalyn makes maximum earnings of \$164 per week by tutoring 8 hours and delivering pizza 12 hours.